

INTERACTION BETWEEN AXIAL AND ANNULAR JETS LEAVING A CYLINDRICAL CAVITY AND AN INCOMING SUPERSONIC GAS FLOW

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Many researchers have studied pressure pulsations that arise in jets, separated flows, and bottom flows. Certain investigations devoted to oscillatory processes in Hartmann resonators were discussed in [1]. Such processes have been modeled by the method of coarse particles, the Chudov–Roslyakov finite-difference scheme, the Godunov–Kolgan scheme, and other methods.

In an experiment conducted in [2], stable pressure pulses were obtained in a supersonic flow past a hollow cylindrical body (cylinder) with its open end facing the incoming flow. In [3], the experimental data obtained in [2] was compared with results calculated on the basis of kinetically consistent difference schemes. Good agreement was obtained with the experimental data within the region of Reynolds numbers $Re_\infty \geq 10^5$, where the effect of Re_∞ on the main characteristics of the process (mean shock-wave decay, amplitude of shock-wave pulsations, period of oscillation, standard deviations of the pressure pulsations) is negligible.

Here, we use the Godunov method to examine the problem of the supersonic flow of an inviscid gas past hollow cylindrical bodies. It is shown numerically that it is possible to control oscillatory flow regimes by the injection of gas from the bottom of the cavity.

1. We will examine the flow of an ideal gas with the Mach number $M_\infty = 3.7$ about a cylinder (Fig. 1). The geometric characteristics of the cylinder ($l/D = 1.6$, $\delta/D = 0.04$) are the same as in [2, 3].

The equations of gas dynamics are as follows in the cylindrical coordinate system [4]

$$\begin{aligned} \frac{\partial \rho r}{\partial t} + \frac{\partial \rho u r}{\partial x} + \frac{\partial \rho v r}{\partial r} &= 0, \\ \frac{\partial \rho u r}{\partial t} + \frac{\partial (p + \rho u^2) r}{\partial x} + \frac{\partial \rho u v r}{\partial r} &= 0, \\ \frac{\partial \rho v r}{\partial t} + \frac{\partial \rho u v r}{\partial x} + \frac{\partial (p + \rho v^2) r}{\partial r} &= p, \\ \frac{\partial \rho e r}{\partial t} + \frac{\partial \rho u (e + p/\rho) r}{\partial x} + \frac{\partial \rho v (e + p/\rho) r}{\partial r} &= 0, \end{aligned}$$

where p is pressure; ρ is density; u and v are components of the velocity vector along x and r (we assume that the component associated with the angle φ is equal to zero); e is the total energy of a unit mass of the gas; t is time. The system is closed by the equation of state of an ideal gas.

The quantities were converted to dimensionless form as follows:

$$\begin{aligned} r &= \bar{r}D/2, \quad x = \bar{x}D/2, \quad t = \bar{t}D/2a_\infty, \quad a = \bar{a}a_\infty, \\ u &= \bar{u}a_\infty, \quad v = \bar{v}a_\infty, \quad \rho = \bar{\rho}\rho_\infty, \quad p = \bar{p}\rho_\infty a_\infty^2 \end{aligned}$$

(a_∞ is sonic velocity in the incoming flow and D is the diameter of the cylinder (see Fig. 1)).

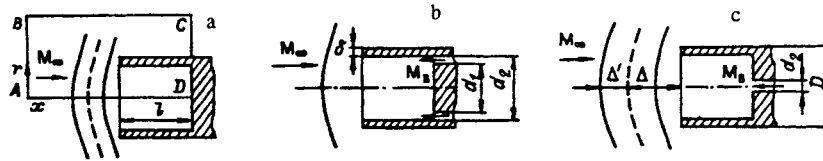


Fig. 1

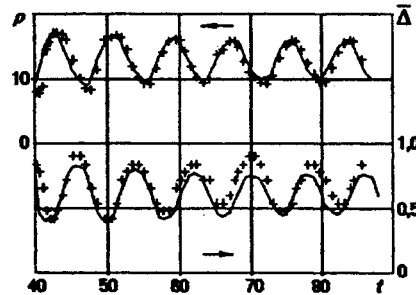


Fig. 2

As the initial data, we used dimensionless parameters of the undisturbed incoming flow:

$$p = p_\infty = 1/\gamma, \quad \rho = \rho_\infty = 1, \quad u = u_\infty = M_\infty, \quad v = 0$$

(γ is the adiabatic constant, $\gamma = 1.14$ in the calculations). Here and below, we drop the bars above the dimensionless quantities r , x , t , a , u , v , ρ , and p .

As the boundary conditions, we adopted the condition of nonflow on the surface of the body and the conditions characterizing the incoming flow. The rectangular region ABCD (Fig. 1a) included the immediate neighborhood of the body in the meridional section. The parameters of the incoming flow were assigned on side AB upstream of the body (the "inlet boundary"). Conditions corresponding to continuous zeroth-order continuation ("smooth conditions") were assigned on the other sections of the boundary of the theoretical region.

The problem was solved by the method of decay of an arbitrary discontinuity [4]. The through computing scheme made it possible to avoid the problems connected with having to identify the surfaces of discontinuity and satisfy compatibility conditions on these surfaces.

2. Calculations were performed on rectangular grids with dimensions ranging from 60×40 to 100×40 . The distribution of the grid nodes within the cavity was uniform, but the ratio of the dimensions of the sides of the cells was varied for different grids: $\Delta r/\Delta x = 1-0.4$. The increments of x and r increased with increasing distance from the surface of the cylinder, allowing us to correctly choose the boundary of the calculated region (with allowance for influence regions).

In all of the variants, we obtained pressure pulsations on the cavity bottom whose amplitudes were closely related to the amplitude of the shock wave in front of the edge of the cylinder $\Delta^* = \Delta'/D$ (see Fig. 1). The same phenomenon was observed in [2, 3]. The Strouhal number $Sh = s/a_0 t^0$, calculated on different grids, was within the range $Sh = 0.23-0.26$ ($Sh \approx 0.25$ in the experiment in [1], $Sh = 0.246$ in the calculation in [3]). Here, t^0 is the period of oscillation; a_0 is sonic velocity at the stagnation temperature; $s = (l + \Delta)$ is the characteristic length (see Fig. 1).

The mean decay of the shock wave $\Delta^0 = \Delta/D$ and the amplitude of the pulsations increased with a decrease in the ratio of the sides of the grid cells $\Delta r/\Delta x$ inside the cavity, which can probably be attributed to the effect of approximate viscosity. In practice, this effect is manifest only in regions with large gradients: on the shock wave, near the surface of the body, in a flow separation region, etc. In this case, the coefficient of system viscosity (and, thus, the width of the resulting "diffuse" shock wave) depends on local flow velocity and cell size [5]. In the experiment in [2], a sharp increase in the mean decay of the wave and the amplitude of the pulsations was seen with a decrease in Re_∞ in the neighborhood $Re_\infty = 10^5$.

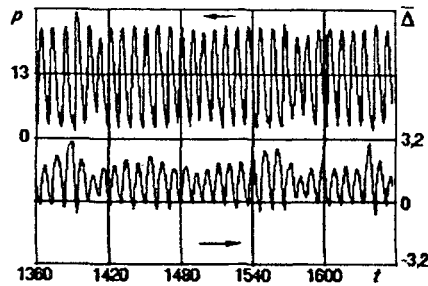


Fig. 3

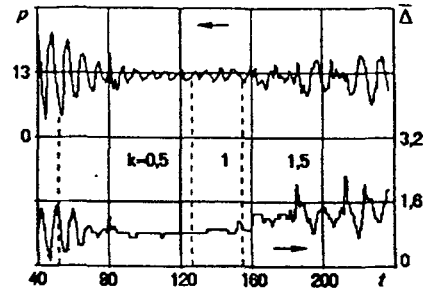


Fig. 4

Comparison of our results with the results in [2] showed that the main pulsation characteristics obtained from calculations on a 100×40 grid ($\Delta r/\Delta x = 1$) ($Sh = 0.252$, $\Delta^0 = 0.33$, $\Delta^* = 0.12$, $\sigma^0 = \sigma/p_0' = 0.2$), agree well with [2] for $Re_\infty > 10^5$. Here, σ represents the standard deviations of the pressure pulsations on the bottom of the cylinder; p_0' is the total pressure behind the normal shock wave. The values $Sh = 0.237$, $\Delta^0 = 0.4$, and $\Delta^* = 0.3$ on the 100×40 grid ($\Delta r/\Delta x = 0.4$) are close to the experimental data for $Re_\infty = 5 \cdot 10^4$ (values of σ were presented in [2] only for $Re_\infty > 10^5$). It should be noted that the values calculated in [3] for $Re_\infty = 5 \cdot 10^4$ ($\Delta^0 = 0.3$ and $\Delta^* = 0.08$) agree with the experimental data reported in [1] for $Re_\infty \geq 10^5$. Figure 2 shows the change in pressure at the center of the bottom of the cavity and the decay of the shock wave from the cylinder edge $\bar{\Delta}$ (the decay was referred to the radius of the cylinder). It is evident from Fig. 2 that the calculated results (+'s) obtained on a 100×40 grid ($\Delta r/\Delta x = 1$) agree satisfactorily with the data from [3] (lines).

3. The velocity vector in the cavity periodically changes direction during the pulsations, as noted in [3]. To explore the feasibility of controlling this process, we conducted a series of calculations in the presence of injection from the bottom of the cylinder. As was shown in [5] in the solution of a similar problem (involving supersonic flow past a cylindrical end at $M_\infty = 3.5$ with the injection of a sonic jet from an annular slit counter to the flow and parallel to axis of symmetry), Euler's equations for unsteady conditions can be used in the given case, since turbulent exchange plays the main role in the regions in which the jet mixes with the incoming flow. At the same time, according to the calculations and comparisons with [1, 5], the effect of approximate viscosity is similar to the effect of the transport properties of a turbulent flow. This similarity can be said to exist because the properties of the resulting numerical flow are qualitatively similar to the phenomena observed in a turbulent flow with $Re = 10^5$ - 10^7 . Thus, the flows with injection considered in this investigation can also be studied by constructing the numerical model on the basis of Euler's nonsteady equations with an approximate dissipation mechanism that leads naturally to the structure of the approximation itself [5].

In the presence of injection, the above-described formulation of the problem of external flow on the body must be supplemented by assigning conditions that express the discharge of the jet at the sonic velocity $M_d = 1$. For this, the parameters of the jet were assigned to the corresponding cells of the grid on the surface of the body. We examined the injection of an annular wall jet with $d_1/D = 0.84$ and $d_2/D = 0.92$ (Fig. 1b) and an axial jet with $d_1/D = 0$ and $d_2/D = 0.08$ (Fig. 1c). Injection rate $k = \rho_d u_d^2 / \rho_\infty u_\infty^2$ was varied within the range $k = 0.1$ - 1.5 . The parameters of the jet in the calculations whose results are shown below were as follows: $M_d = 1$, $u_d = -1$, $v_d = 0$, $\rho_d = k M_\infty^2$, $\gamma_d = 1.4$. As the initial data, we used the field of gasdynamic characteristics obtained in a calculation performed without injection for the moment of time $t = 51$.

The calculations showed that the injection of low-intensity axial jets into the cylinder produces more energetic pulsations than in the absence of injection. Figure 3 shows the change in pressure at the center of the bottom of the cylinder and the decay of the shock wave with the injection of an axial jet of intensity $k = 0.1$. It is apparent that the changes in these parameters take the form of increasing and decreasing oscillations with the period $t_1 \approx 80$. We see that the wave periodically ($t_2 \approx 240$) decays particularly rapidly, with $\Delta^* \approx 0.8$. The latter value is twice as great as the amplitude of the wave pulsations obtained on the same grid without injection.

An increase in the rate of injection of the axial jet ($k = 0.25$) is accompanied by a sharp increase in the amplitude of the shock (by an average factor of two compared to the variant without injection). At peak levels of decay, repeating with the period $t \approx 650$ (calculations were performed up to $t \approx 2000$), the amplitude was commensurate with the length of the cylinder $\Delta^* \approx 2$. After these peaks, the pulsations in the cylinder decreased over the period $t \approx 160$ (the velocity vector in

the wall region was oriented counter to the incoming flow). The pulsations were subsequently reestablished over a period $t \approx 80$ (the velocity vector in the wall region again began to periodically change directions).

The injection of an annular jet (Fig. 1b) orients the velocity vector counter to the incoming flow in the wall region of the cavity. Decay of the pulsations is seen with an injection rate $k = 0.5$ (Fig. 4). The flow remains steady up to $k \approx 1$, while decay of the shock wave increases somewhat ($\Delta^0 \approx 0.55$). A further increase in injection rate ($k = 1.5$) leads to interaction of the jet with the shock wave, causing the flow pattern to become distinctly unsteady in character.

Comparison of the data calculated with the use of different grids showed good qualitative agreement. There were some differences in the amplitudes of the pulsations and mean shock decay. As noted previously, these differences are related to the effect of approximate viscosity, which depends both on the structure of the flow and on the geometry of the grid. The results shown in Figs. 3 and 4 were obtained on a 100×40 grid ($\Delta r/\Delta x = 0.4$).

Thus, as the calculations showed, it is possible to control the pulsation process by regulating injection rate and the location of the injected jet on the bottom of the cavity.

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